Exercises

Communicate

1. Compare the natural and exponential logarithmic functions with the base-10 exponential and logarithmic functions.
2. Give a real-world example of an exponential growth function and of an exponential decay function that each have the base e.
3. State the continuous compounding formula, and describe what each variable represents.
4. Describe how the continuous compounding formula can represent continuous growth as well as continuous decay.

Guided Skills Practice

APPLICATION

Evaluate $f(x) = e^x$ to the nearest thousandth for each value of x. *(EXAMPLE 1)*

5. $x = 3$
6. $x = 3.5$
7. **INVESTMENTS** An investment of $1500 earns an annual interest rate of 8.2%. Compare the final amounts after 5 years for interest compounded quarterly and for interest compounded continuously. *(EXAMPLE 2)*

Evaluate $f(x) = \ln x$ to the nearest thousandth for each value of x. ***(EXAMPLE 3)***

8. $x = 5$ **9.** $x = 2.5$

10. INVESTMENTS How long does it take an investment to double at an annual interest rate of 7.5% compounded continuously? ***(EXAMPLE 4)***

11. ARCHAEOLOGY A piece of charcoal from an ancient campsite is found in an archaeological dig. It contains 9% of its original amount of carbon-14. Estimate the age of the charcoal. ***(EXAMPLE 5)***

Practice and Apply

Evaluate each expression to the nearest thousandth. If the expression is undefined, write *undefined*.

12. e^6 **13.** e^9 **14.** $e^{1.2}$ **15.** $e^{3.4}$

16. $2e^{0.3}$ **17.** $3e^{0.05}$ **18.** $2e^{-0.5}$ **19.** $3e^{-0.257}$

20. $e^{\sqrt{2}}$ **21.** $e^{\frac{1}{4}}$ **22.** $\ln 3$ **23.** $\ln 7$

24. $\ln 10{,}002$ **25.** $\ln 99{,}999$ **26.** $\ln 0.004$ **27.** $\ln 0.994$

28. $\ln \frac{1}{5}$ **29.** $\ln \sqrt{5}$ **30.** $\ln(-2)$ **31.** $\ln(-3)$

For Exercises 32–35, write the expressions in ascending order.

32. $e^2, e^5, \ln 2, \ln 5$ **33.** $e, e^0, \ln 1, \ln \frac{1}{2}$

34. $e^{2.5}, \ln 2.5, 10^{2.5}, \log 2.5$ **35.** $e^{1.3}, \ln 1.3, 10^{1.3}, \log 1.3$

State whether each equation is always true, sometimes true, or never true.

36. $e^{5x} \cdot e^5 = e^{15x}$ **37.** $(e^{4x})^3 = e^{12x}$ **38.** $e^{6x-4} = e^{6x} \cdot e^{-4}$ **39.** $\frac{e^{5x}}{e^4} = e^{2x}$

Simplify each expression.

40. $e^{\ln 2}$ **41.** $e^{\ln 5}$ **42.** $e^{3 \ln 2}$ **43.** $e^{2 \ln 5}$

44. $\ln e^3$ **45.** $\ln e^4$ **46.** $3 \ln e^2$ **47.** $2 \ln e^4$

Write an equivalent exponential or logarithmic equation.

48. $e^x = 30$ **49.** $e^x = 1$ **50.** $\ln 2 \approx 0.69$

51. $\ln 5 \approx 1.61$ **52.** $e^1 \approx 1.40$ **53.** $e^{0.69} \approx 1.99$

Solve each equation for x by using the natural logarithm function. Round your answers to the nearest hundredth.

54. $35^x = 30$ **55.** $1.3^x = 8$ **56.** $3^{-3x} = 17$

57. $36^{2x} = 20$ **58.** $0.42^{-x} = 7$ **59.** $2^{-\frac{1}{3}x} = 10$

60. Sketch $f(x) = e^x$ for $-1 \le x \le 2$. A line that intersects a curve at only one point is called a *tangent line* of the curve.

a. Sketch lines that are tangent to the graph of $f(x) = e^x$ at $x = 0.5$, $x = 0$, $x = 1$, and $x = 2$.

b. Find the approximate slope of each tangent line. Compare the slope of each tangent line with the corresponding y-coordinate of the point where the tangent line intersects the graph.

c. Make a conjecture about the slope of $f(x) = e^x$ as x increases.