

Multiplying Complex Numbers

Recall FOIL: $(a + b)(c + d) = ac + ad + bc + bd$
 $i^2 = -1$

I. Model Problems

In these examples we will multiply complex numbers

Example 1: Simplify $2i(3 + 7i)$

Distribute $2i$.

$$\begin{aligned} & 2i(3 + 7i) \\ & 2i(3) + 2i(7i) \\ & 6i + 14i^2 \\ & 6i + 14(-1) \\ & 6i - 14 \\ & -14 + 6i \end{aligned}$$

Substitute -1 for i^2 . Simplify.

Rewrite in standard form.

Answer: $2i(3 + 7i) = -14 + 6i$

Example 2: Simplify $(5 + 2i)(6 - 4i)$

Multiply with the FOIL method.

$$\begin{aligned} & (5 + 2i)(6 - 4i) \\ & 5(6) + 5(-4i) + 2i(6) + 2i(-4i) \\ & 30 - 20i + 12i - 8i^2 \\ & 30 - 8i - 8i^2 \\ & 30 - 8i - 8(-1) \\ & 30 - 8i + 8 \\ & 38 - 8i \end{aligned}$$

Substitute -1 for i^2 . Simplify.

Answer: $(5 + 2i)(6 - 4i) = 38 - 8i$

II. Practice Problems

Simplify.

1. $6i(4 - 12i)$

2. $-3i(9 - 6i)$

3. $-11i(3 + 9i)$

4. $2.4i(18 + 12i)$

5. $-0.2i(16 + 8i)$

6. $(2 + 4i)(3 + 3i)$

7. $(3 + 5i)(4 + 4i)$

8. $(2 - 4i)(3 + 5i)$

9. $(-3 + 2i)(-3 - 4i)$

10. $(5 - 10i)(3 + 5i)$

11. $(-3 - 7i)(-12 - 2i)$

12. $(15 + 3i)(4 - 15i)$

13. $3i(4 + 2i)(2 + 5i)$

14. $-2i(3 - 7i)(4 + 2i)$

15. $2i(4 - 5i)(4 + 5i)$

16. $10(3 - 2i)(4 + 3i)$

17. $6i(7 - 10i)(7 + 10i)$

18. $(3 + 2i)^2$

19. $-(2 + 2i)^2$

20. $i(4 + 3i)^2$

III. Recognizing a Pattern

Multiply the next few complex number pairs and try to recognize a pattern

21) $(3 + 2i)(3 - 2i)$

22) $(4 + 5i)(4 - 5i)$

23) $(7 - 3i)(7 + 3i)$

24) Look carefully at questions 21-23. These complex numbers are known as complex conjugates. Explain why complex conjugates do **not** have an “i” term

Challenge Problems

1. Simplify $(a - bi)(a + bi)$

2. Find the error in the student’s work.

$$\begin{aligned} &(6 - 2i)(8 + 4i) \\ &48 + 24i - 16i - 8i^2 \\ &48 + 8i - 8 \\ &40 + 8i \end{aligned}$$

3. If $(4 + ai)(4 - bi) = 16 + ab$ what must be true of a and b .

4. Find a if $(1 + ai)(1 - ai) = 2$.

5. Find the error in the student’s work.

$$\begin{aligned} &(3 + 6i)(2 - 5i) \\ &6 - 30i \end{aligned}$$